

## **THERMODYNAMICS IN $n$ -SPATIAL DIMENSIONS**

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### **Abstract**

Conventional thermodynamics is well established, and is usually applied to three-dimensional space. Thermodynamics has an impact on astrophysical phenomena through conventional physics including the general theory of relativity. Accordingly, extensions of thermodynamics to multiple dimensions may also have astrophysical implications including the understanding of black holes dynamics, inflation following the big bang, and the evolution of the universe. This paper reviews the extension of thermodynamics to multiple spatial dimensions by reviewing a number of basic concepts including black body radiation; internal energy, heat capacity, entropy; and the ratio of specific heats ( $C_p/C_v$ ).

**KEYWORDS:** Thermodynamics in  $n$ -spatial dimensions; astrophysical implications; black body radiation; ratio of specific heats ( $C_p/C_v$ ); and internal energy, heat capacity, and entropy.

## 1.0 Introduction

Thermodynamics and statistical mechanics in conventional spacetime (three spatial dimensions ( $n$ ) and a time dimension) are well defined. For example, Newtonian mechanics and general relativity yield orbits with specific characteristics that are observed in physical structures including the Solar system. In a previous paper, Newtonian orbital results were generalized to  $n$ -dimensions ( $n > 3$ ) with results that deviated from conventional orbits and associated characteristics [1].

In a similar manner,  $n$ -dimensional thermodynamics could yield results that differ from conventional thermodynamics and statistical mechanics [2-9]. A portion of these possible differences are addressed in this paper with an emphasis on the temperature dependence of selected physical quantities.

As an initial investigation of multidimensional thermodynamics, selected thermodynamic relationships are evaluated to determine their  $n$ -dimensional counterparts. For example, in conventional spacetime, (1) the energy density of black body radiation depends on the absolute temperature ( $T$ ) that varies as  $T^4$ , (2) the specific heat of solids varies as  $T^3$ , and (3) the specific heat ratio ( $C_p/C_v$ ) for a monatomic ideal gas is  $5/3$  [2-9].

Additional aspects of conventional thermodynamics and its applications are provided by Santilli [10-11]. Specific thermodynamic applications of Santilli's approach [10-11] are provided in [12].

In general, these thermodynamic relationships could depend on the dimensionality of spacetime, particularly the number of spatial dimensions. The generalized  $n$ -dimensional thermodynamic relationships, including statistical mechanics, have potential astrophysical importance associated with black hole thermodynamics, entropy evolution following the big bang, and inflation.

## 2.0 Assumptions

Given the theoretical nature of this paper, the assessment of the effects of multiple dimensions on selected thermodynamic quantities must rely on a number of assumptions. These assumptions include, but are not limited to:

1. Thermodynamic relationships in multiple dimensions have a similar physical form as their conventional spacetime counterparts, and must yield the expected three-dimensional results when  $n = 3$ .
2. The extra dimensions do not interact with each other or with conventional spacetime.
3. Specific characteristics of the extra dimensions are not considered.

4. The scale of the extra dimensions is not specified.
5. The specific spacetime topology, including the character of any compactified dimensions, evolves in a manner that is consistent with the aforementioned assumptions.
6. Any interaction of astrophysical phenomena (e.g., black body radiation) with the extra dimensions that could affect the character of these dimensions is not considered.
7. The number of polarization states allowed for photons in  $n$  dimensions is unknown. This uncertainty is absorbed into the  $C(n)$  term in (8).
8. All dimensions are independent.

The aforementioned assumptions are utilized in subsequent discussion.

### 3.0 Thermodynamic Relationships

In order to assess the theoretical effects of the postulated extra dimensions, specific thermodynamic quantities are evaluated. These include (1) black body radiation, (2) internal energy, heat capacity, and entropy, and (3)  $C_p/C_v$  ratio.

#### 3.1 Black Body Radiation

In conventional spacetime, the energy density of blackbody radiation depends on temperature. The energy density of the output of a blackbody radiator scales as the absolute temperature ( $T^4$ ). A general relationship for extending conventional thermodynamics to multiple dimensions must rely on a number of assumptions noted previously.

The first and second laws of thermodynamics leads to an internal energy equation [2]

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P \quad (1)$$

where the parameters in Eq. 1 refer to the blackbody radiation. In Eq. 1,  $U$  is the total internal energy,  $V$  is the volume,  $T$  is the absolute temperature, and  $P$  is the pressure. The total energy is related to the energy density ( $\rho$ )

$$U = \rho V \quad (2)$$

In conventional spacetime, the pressure of blackbody radiation is related to the density [8]



$$P = \rho/3 \quad (3)$$

In multiple dimensional spacetime, Eq. 3 will in general depend on the number of spatial dimensions. In this paper, it is additionally assumed that the blackbody radiation in  $n$  dimensions follows from the conventional spacetime relationship of (3)

$$P = \rho/n \quad (4)$$

(1), (2), and (4) lead to the general result for  $n$ -dimensional spacetime

$$\rho = T \frac{1}{n} \frac{d\rho}{dT} - \frac{\rho}{n} \quad (5)$$

Eq. 5 simplifies to the result

$$\frac{d\rho}{\rho} = (n + 1) \frac{dT}{T} \quad (6)$$

that has the solution that can be written in the form

$$\rho = CT^{n+1} \quad (7)$$

where  $C$  is a constant of integration. A similar result can be derived in an alternative manner utilizing statistical mechanics [13,14]. (7) leads to the expected result that in three spatial dimensions the black body radiation intensity varies as  $T^4$ .

### 3.2 Internal Energy, Heat Capacity, and Entropy

In the previous section, a thermodynamic approach utilized the internal energy to address black body radiation. This section utilizes a statistical mechanics approach to perform an internal energy calculation, and to extend the results to determine of the heat capacity and entropy in  $n$ -dimensions. This is accomplished by generalizing conventional thermodynamics and statistical mechanics. Assuming one state per unit lattice in  $n$ -dimensional  $k$ -space  $(2\pi/L)^n$  for each polarization, the density of states is

$$D(\omega)d\omega = \left[ \frac{(2\pi)^{n/2}}{\Gamma(\frac{n}{2})} \right] \left( \frac{L}{2\pi} \right)^n \omega^{n-1} dk = C(n) \omega^{n-1} d\omega \quad (8)$$



where the  $C(n)$  term includes the volume of an  $n-1$  dimensional sphere, the number of states per unit lattice in  $k$ -space, and any polarization terms that involve an assumed linear dispersion relationship  $\omega = v k$ . The dispersion relationship relates the wave vector ( $k$ ), velocity ( $v$ ), and angular frequency ( $\omega$ ).

The total internal energy is defined by the density of states

$$U = \int D(\omega) d\omega \hbar \omega \langle n(\omega) \rangle = \int d\omega \hbar C(n) \omega^n \left( \frac{1}{\exp(\hbar \omega / kT) - 1} \right) \quad (9)$$

where  $k$  is the Boltzmann constant, and  $\langle n(\omega) \rangle$  is the average energy as a function of frequency. Using a change of variables  $x = \hbar \omega / kT$ , leads to a simplified result for the internal energy

$$U = B(n) T^{n+1} \int \left( \frac{x^n}{e^x - 1} \right) dx \quad (10)$$

where the constant  $B(n)$  is related to  $C(n)$  multiplied by the terms generated by the change of variables and the constants appearing in (9). (10) reduces to the expected three dimensional result that was also derived in Section 3.1.

In conventional spacetime, (10) is usually evaluated using the Debye approximation where the upper integration limit is the Debye frequency. This approximation may not be valid in an  $n$  dimensional space. Instead, the integrals of (9) and (10) are evaluated using an unrestricted upper integration limit

$$U = B(n) T^{n+1} \int_0^\infty \left( \frac{x^n}{e^x - 1} \right) dx \quad (11)$$

(11) has the solution

$$U = B(n) T^{n+1} \Gamma(n+1) \zeta(n+1) = J(n) T^{n+1} \quad (12)$$

where  $\Gamma(n)$  is a gamma function of order  $n$ ,  $\zeta(n)$  is a Riemann zeta function of order  $n$ , and  $J(n) = B(n) \Gamma(n+1) \zeta(n+1)$ .

(12) permits the calculation of the heat capacity ( $C$ ) using standard thermodynamic relations

$$C = \frac{\partial U}{\partial T} = J(n) (n+1) T^n \quad (13)$$

In a similar manner, the entropy (S) is

$$S = \int \frac{C}{T} dT = J(n) \frac{n+1}{n} T^n + a \quad (14)$$

where a is a constant of integration that is to be determined based on observations.

### 3.3 Cp/Cv Ratio

The equipartition theorem suggests that the specific heat at constant volume ( $C_v$ ) is

$$C_v = \frac{dg}{2} \quad (15)$$

where d is the number of degrees of freedom for the species and g is a constant. For a monatomic molecule in an n-dimensional space  $d = n$ . Accordingly,

$$C_v = \frac{ng}{2} \quad (16)$$

In addition, the heat capacities at constant volume and constant pressure ( $C_p$ ) are related by the relationship [3]

$$C_p = C_v + g \quad (17)$$

(14) and (15) lead to the result

$$\frac{C_p}{C_v} = \frac{n+2}{n} \quad (18)$$

(18) provides the expected result of 5/3 for conventional spacetime.

## 4.0 Discussion

Thermodynamics has a significant impact on astrophysical phenomena [15-19]. For example, Hawking notes that there is a strong relationship between black holes and thermodynamics [17]. The commonly derived relationships involving information flow, heat capacity, Hawking radiation, and entropy could require modification in an n-dimensional spacetime. Relevant conclusions regarding the

behavior of thermodynamic quantities associated with black holes could be modified if multiple dimensional spacetime emerges in the vicinity of these bodies [15-19].

Hawking [17] noted that black holes have negative specific heat. This result may change as a function of the dimensionality of spacetime, and the interaction of these dimensions with their environment. In addition, derived relationships based on conventional spacetime (e.g., thermal equilibrium or time reversibility) may also require modification. These considerations could affect current theoretical studies of black holes and their relationship to white holes, and assumed time-symmetric behavior.

Current results related to a variety of topics including entropy, information flow, and Hawking radiation should also be reexamined for the theoretical consequences in multidimensional spacetime. The implications of multidimensional thermodynamics on quantum field theory, quantum gravity, and general relativity are also uncertain and should be carefully evaluated.

## 5.0 Conclusions

The generalization of thermodynamics to  $n$  spatial dimensions results in relationships that are consistent with their three-dimensional counterparts. Specific relationships are provided for blackbody radiation, internal energy, heat capacity, entropy, and  $C_p/C_v$  ratio. These and other thermodynamic relationships when generalized to  $n$  spatial dimensions could have astrophysical significance associated with the big bang, inflation, black hole thermodynamics, and entropy flow.

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